نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.1** Use the differential equation approach to find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.1 and plot the response including the time interval just prior to switch action.

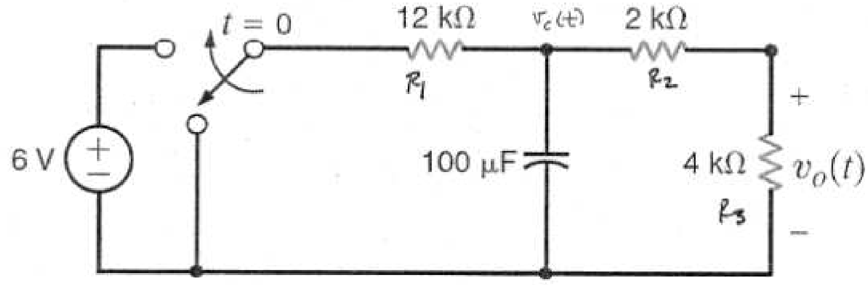



***Figure P7.1***

---

SOLUTION:

$v_c(0^-) = 0V$ $\quad$ for $t>0$: $\quad \dfrac{6-v_c}{R_1} + \dfrac{v_o - v_c}{R_2} - C\dfrac{dv_c}{dt} = 0 \qquad v_o = \dfrac{v_c R_3}{R_2+R_3} = \alpha v_c$

Multiply by $\alpha \Rightarrow \quad \dfrac{6\alpha}{R_1} + \dfrac{v_o}{R_2}\left[\alpha - 1\right] - \dfrac{v_o}{R_1} - \dfrac{C\,dv_o}{dt} = 0$

$$\frac{dv_o}{dt} + v_o\left[\frac{1}{R_1C} + \frac{1-\alpha}{R_2C}\right] - \frac{6\alpha}{R_1C} = 0 \qquad \text{let } \frac{1}{R_1C} + \frac{1-\alpha}{R_2C} = B$$

assume $\quad v_o(t) = K_1 + K_2 e^{-t/\tau}$

$$-\frac{K_2}{\tau}e^{-t/\tau} + K_1B + K_2Be^{-t/\tau} - \frac{6\alpha}{R_1C} = 0 \qquad \begin{cases} \tau = 1/B = C\left[\dfrac{R_1(R_2+R_3)}{R_1+R_2+R_3}\right] \\ K_1 = \dfrac{6\alpha}{R_1BC} = \dfrac{6R_3}{R_1+R_2+R_3} \end{cases}$$

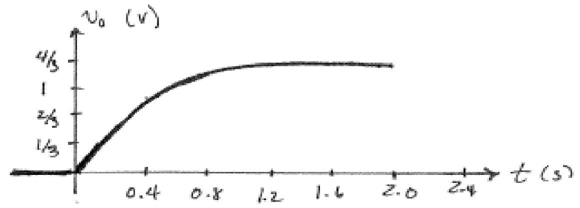
$\tau = 0.4s \qquad K_1 = 1.33V$

$v_o(0) = v_c(0)\,\alpha = 0 = K_1 + K_2 \Rightarrow K_2 = -1.33V$

$$\boxed{v_o(t) = 1.33 - 1.33e^{-2.5t}\ V}$$

$\underline{t = 0^+}$: $\quad v_c(0^+) = 0 \qquad v_o(0^+) = \dfrac{6R_3}{R_1+R_2+R_3} = 1.33V$

$\underline{t = 0^-}$ $\quad v_o(t) = 0$



## نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.2** Use the differential equation approach to find $v_C(t)$ for $t > 0$ in the circuit in Fig. P7.2 and plot the response including the time interval just prior to closing the switch.

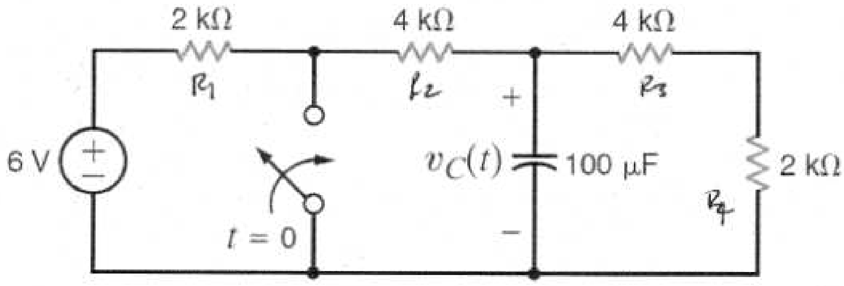



*Figure P7.2*

---

**SOLUTION:** $v_C(0^+) = v_C(0^-) = \dfrac{6\,(R_3+R_4)}{R_1+R_2+R_3+R_4} = 3V$

for $t > 0$: $\dfrac{v_C}{R_2} + \dfrac{v_C}{R_3+R_4} + C\dfrac{dv_C}{dt} = 0$ let $v_C(t) = K_1 + K_2 e^{-t/\tau}$

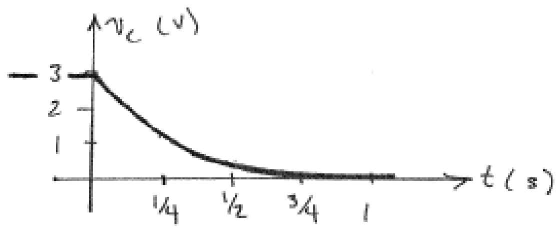
$$K_1\left(\frac{1}{R_2}+\frac{1}{R_3+R_4}\right) + K_2\left(\frac{1}{R_2}+\frac{1}{R_3+R_4}\right)e^{-t/\tau} - \frac{K_2 C}{\tau}e^{+t/\tau} = 0$$

yields $K_1 = 0$ $\quad \tau = C\left\{\dfrac{R_2(R_3+R_4)}{R_2+R_3+R_4}\right\} = 0.24s$

$v_C(0^+) = 3 = K_1 + K_2 \Rightarrow K_2 = 3V$

$$\boxed{v_C(t) = 3e^{-t/0.24} \text{ V}}$$

for $t = 0^-$: $v_C(0^-) = 3V$



نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.5** Use the differential equation approach to find $v_C(t)$ for $t > 0$ in the circuit in Fig. P7.5 and plot the response including the time interval just prior to opening the switch. CS

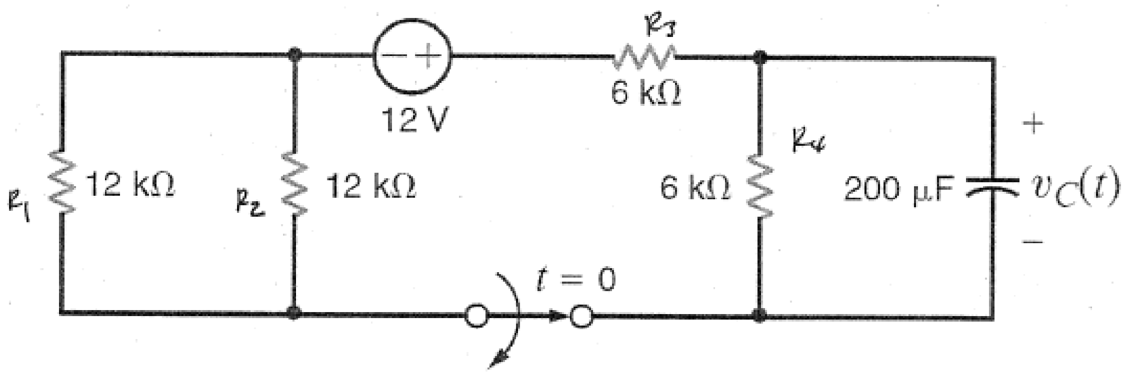



*Figure P7.5*

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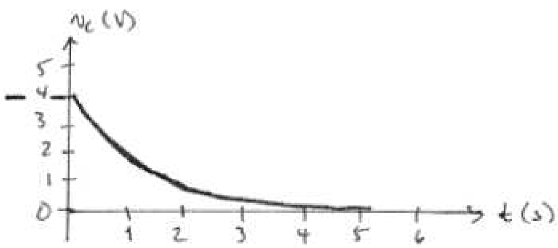
SOLUTION: $v_c(0^-) = v_c(0^+) = \dfrac{12\,R_4}{R_3+R_4+R_A}$ $\quad R_A = R_1 // R_2 = 6k\Omega$ $\quad v_c(0^+) = 4V$

for $t > 0$; $\quad \dfrac{v_c}{R_4} + C\dfrac{dv_c}{dt} = 0 \rightarrow \dfrac{dv_c}{dt} + \dfrac{v_c}{R_4 C} = 0$

$v_c(t) = K_1 + K_2 e^{-t/\tau} \quad \Rightarrow \quad \dfrac{dv_c}{dt} + \dfrac{v_c}{\tau} = 0$

$\tau = R_4 C \qquad K_1 = 0 \qquad v_c(0^+) = 4 = K_1 + K_2 \Rightarrow K_2 = 4V$

$$\boxed{v_c(t) = 4e^{-t/1.2}\ V}$$



نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.7** Use the differential equation approach to find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.7 and plot the response including the time interval just prior to closing the switch.

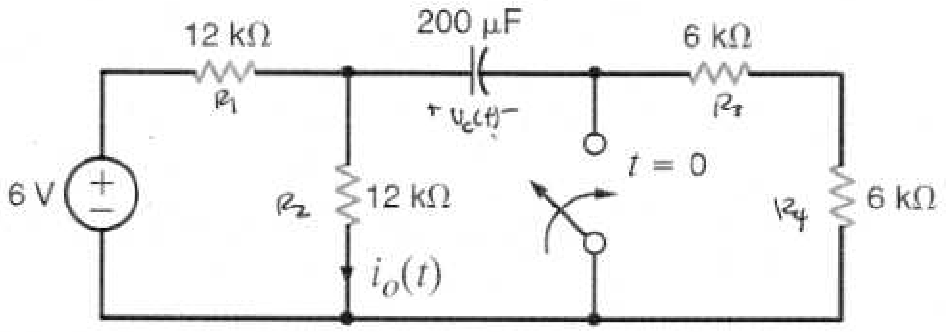



*Figure P7.7*

---

SOLUTION: $v_c(0^-) = v_c(0^+) = \frac{6R_2}{R_1+R_2} = 3V$   $i_o(t) = \frac{v_c(t)}{R_2}$ for $t>0$.

for $t>0$: $\frac{6-v_c}{R_1} = \frac{v_c}{R_2} + C\frac{dv_c}{dt} \Rightarrow \frac{dv_c}{dt} + v_c\left[\frac{1}{R_1C}+\frac{1}{R_2C}\right] - \frac{6}{R_1C} = 0$

Convert to $i_o$: $\frac{di_o}{dt} + i_o\left[\frac{1}{R_1C}+\frac{1}{R_2C}\right] - \frac{6}{R_1R_2C} = 0$

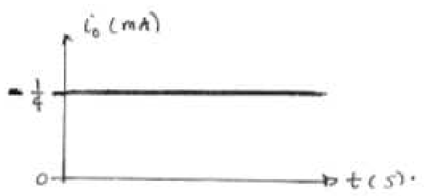
$i_o(t) = K_1 + K_2e^{-t/\tau} \Rightarrow -\frac{K_2}{\tau}e^{-t/\tau} + (K_1+K_2e^{-t/\tau})\left[\frac{1}{R_1C}+\frac{1}{R_2C}\right] - \frac{6}{R_1R_2C} = 0$

yields $\tau = C\frac{R_1R_2}{R_1+R_2} = 1.2s$   $K_1 = \frac{6}{R_1+R_2} = 0.25mA$

$i_o(0^+) = K_1+K_2 = v_c(0^+)/R_2 = 0.25mA \Rightarrow K_2 = 0$

$\boxed{i_o(t) = 0.25mA}$

$t=0^-$: $v_c(0^-) = 3V$   $i_c(0^-) = 0$   $i_o(0^-) = \frac{6}{R_1+R_2} = 0.25mA$



نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.10** Use the differential equation approach to find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.10 and plot the response including the time interval just prior to opening the switch. CS

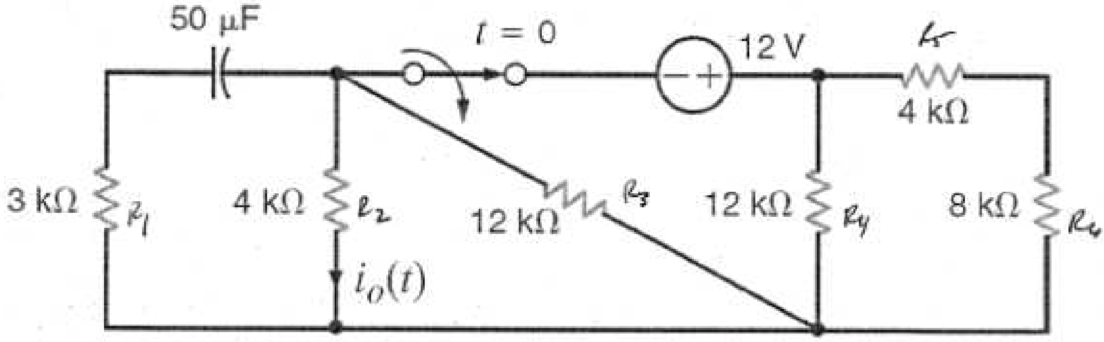
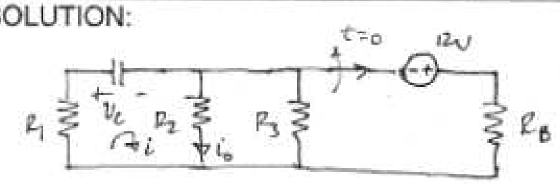



*Figure P7.10*

SOLUTION:




$R_B = R_4 \,//\, (R_5 + R_6) = 6\,k\Omega$

$R_A = R_2 \,//\, R_3 = 3\,k\Omega$

$v_c(0^+) = v_c(0^-) = \frac{12 R_A}{R_A + R_B} = 4V$  $i_o(0^-) = \frac{-v_c(0^-)}{R_2} = -1mA$

For $t>0$, $v_c + iR_A + iR_1 = 0$ & $i = C\frac{dv_c}{dt}$ & $i_o = \frac{R_3}{R_2+R_3}\, i = \alpha i$

yields, $\frac{dv_c}{dt} + \frac{v_c}{C(R_1+R_A)} = 0$

or, $\frac{di_o}{dt} + \frac{i_o}{C(R_1+R_A)} = 0$  where $i_o = K_1 + K_2 e^{-t/\tau}$

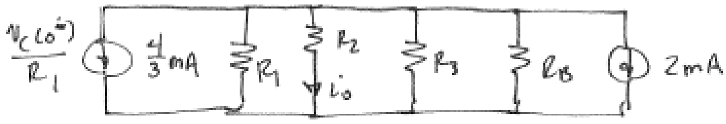
yields, $\tau = C[R_1 + R_A] = 0.3s$  $K_1 = 0$

$i_o(0^+) = \frac{-v_c(0^+)}{R_1 + R_A}\,\frac{R_3}{R_3 + R_2} = -0.5mA = K_1 + K_2 \Rightarrow K_2 = -0.5mA$

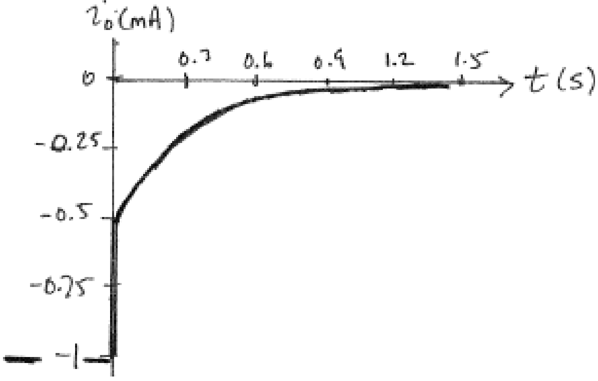
$\boxed{i_o(t) = -0.5\, e^{-t/0.3}\ mA}$

# نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

$t=0^-$:




$$i_o(0^-) = -\frac{(2+4/3)\times10^{-3}\,(1/R_2)}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_B}} = -1\,mA$$



نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.28** Use the step-by-step technique to find $i_o(t)$ for $t > 0$ in the network in Fig. P7.28. CS

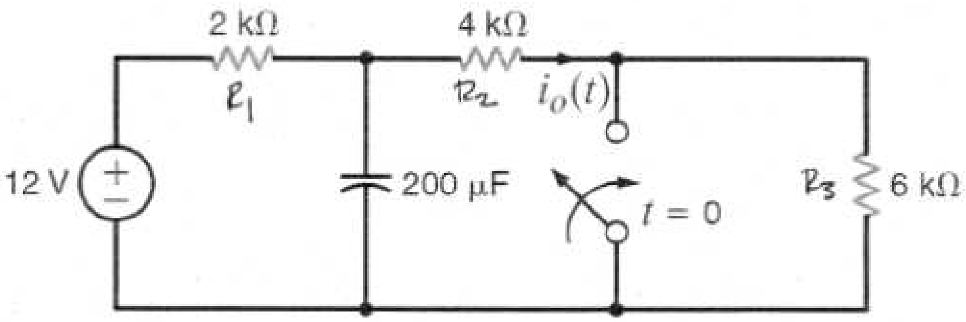
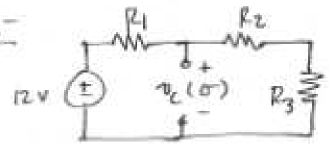



***Figure P7.28***

SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

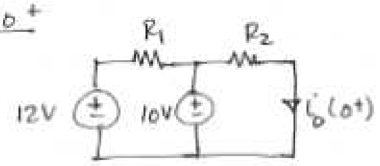
$t = 0^-$




By voltage division: $v_c(0^-) = \dfrac{12(R_2+R_3)}{R_1+R_2+R_3}$

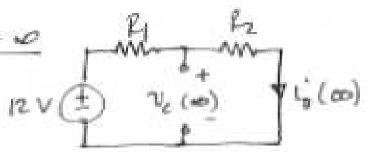
$v_c(0^-) = 10V$

$t = 0^+$



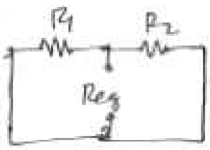

$i_o(0^+) = 10/R_2 = 2.5mA = K_1 + K_2$

$t = \infty$




$i_o(\infty) = \dfrac{12}{R_1+R_2} = 2mA = K_1$

$\tau = ?$




$\tau = C R_{eq}$ $\quad R_{eq} = R_1 // R_2 = \frac{4}{3} k\Omega$

$\tau = 0.267 s$

$$\boxed{i_o(t) = 2 + 0.5 e^{-3.75t} \text{ mA}}$$

# نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.29** Use the step-by-step method to find $v_o(t)$ for $t > 0$ in the network in Fig. P7.29.

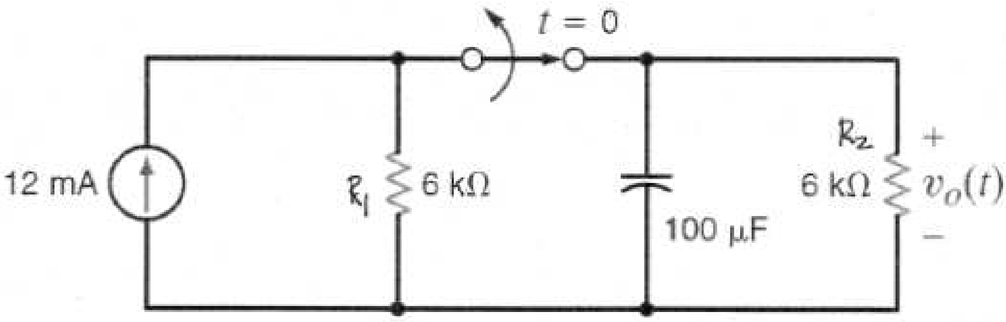



*Figure P7.29*

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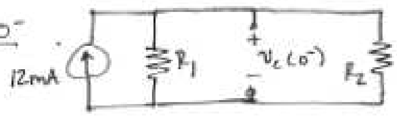
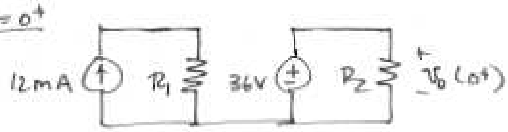
SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$



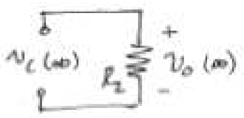

$v_C(0^-) = 12\times10^{-3}\,\dfrac{(R_1R_2)}{R_1+R_2} = 36\text{V}$

$t = 0^+$




$v_o(0^+) = 36 = K_1 + K_2$

$t = \infty$




$v_o(\infty) = 0 = K_1$

$\tau = ?$ $\quad \tau = R_{eq}C \qquad R_{eq} = R_2 = 6k\Omega \qquad \tau = 0.6\,s$

$$\boxed{v_o(t) = 36\, e^{-t/0.6}\ \text{V}}$$

# نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.30** Use the step-by-step method to find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.30.

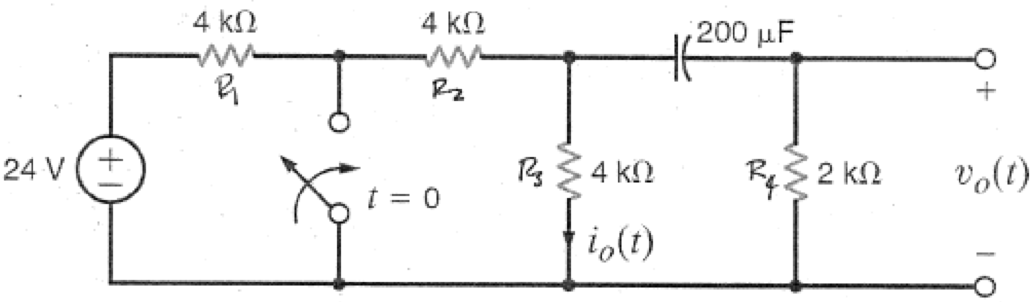



*Figure P7.30*

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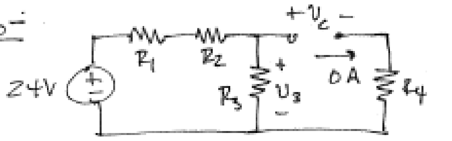
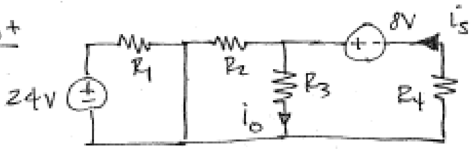
SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$




$v_c = V_3 = \frac{24 R_3}{R_1 + R_2 + R_3} = 8V$

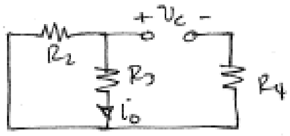
$t = 0^+$




$i_s = \frac{8}{R_4 + R_x}$ $\qquad R_x = \frac{R_2 R_3}{R_2 + R_3} = 2k\Omega$

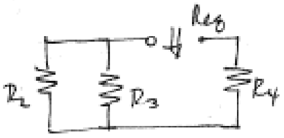
$i_o = \frac{i_s R_2}{R_2 + R_3} = 1mA = K_1 + K_2$

$t = \infty$




$i_o(\infty) = 0 = K_1$

$\tau = ?$




$\tau = R_{eq} C$ $\qquad R_{eq} = \frac{R_2 R_3}{R_2 + R_3} + R_4 = 4k\Omega$

$\tau = 0.8 s$

$$\boxed{i_o(t) = e^{-1.25t} \text{ mA}}$$

# نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

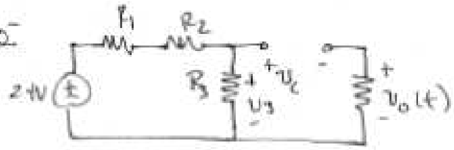
**7.31** Find $v_o(t)$ for $t > 0$ in the network in Fig. P7.30 using the step-by-step technique.

---

**SOLUTION:**

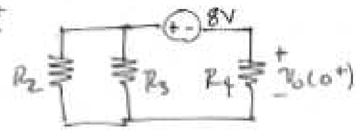
$v_o(t) = K_1 + K_2 e^{-t/\tau}$ $\quad R_1 = R_2 = R_3 = 4k\Omega \quad R_4 = 2k\Omega$

$\underline{t=0^-}$




$v_c = v_3 = \dfrac{24 R_3}{R_1+R_2+R_3} = 8V$

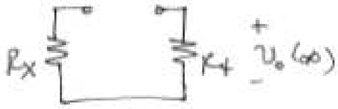
$\underline{t=0^+}$




$R_x = R_2 \,//\, R_3 = 2k\Omega$

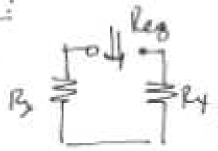
$v_o(0^+) = -\dfrac{8R_4}{R_4+R_x} = -4V = K_1+K_2$

$\underline{t=\infty}$




$v_o(\infty) = 0 = K_1$

$\underline{\tau = ?}$




$\tau = R_{eq} C \quad R_{eq} = R_x + R_4 = 4k\Omega$

$\tau = 0.8s$

$$\boxed{v_o(t) = -4e^{-1.25t}\ V}$$

نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.33** Find $v_o(t)$ for $t > 0$ in the network in Fig. P7.33 using the step-by-step method. CS

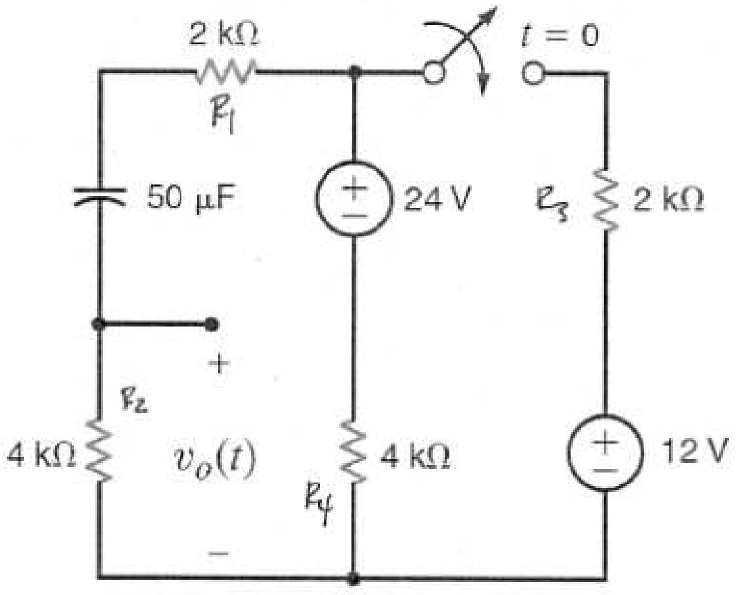
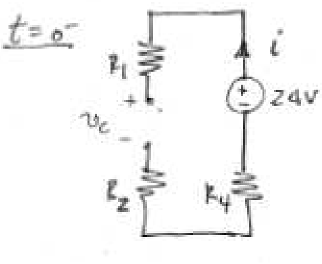



Figure P7.33

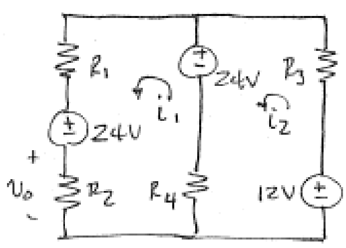
SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$




$i = 0$ so $v_c(0^-) = 24V$

# نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

$t=0^+$




$$24 = i_1(R_1+R_2+R_4) - i_2R_4 + 24$$

or, $\quad i_1(R_1+R_2+R_4) = i_2R_4$

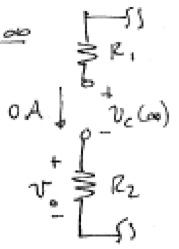
$$12 = i_2(R_3+R_4) + 24 - i_1R_4$$

or $\quad i_1R_4 - i_2(R_3+R_4) = 12$

$i_1 = -\frac{12}{11}\ mA \qquad v_o(0^+) = i_1R_2$

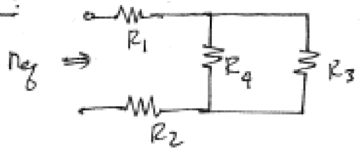
$v_o(0^+) = \frac{48}{11}\ V = K_1 + K_2$

$t=\infty$




$v_o(\infty) = 0 = K_1$

$\tau = ?$




$$R_{eq} = R_1 + R_2 + \frac{R_3R_4}{R_3+R_4} = 7.33\,k\Omega$$

$\tau = 367\ ms$

$$\boxed{v_o(t) = -4.36\,e^{-2.73t}\ V}$$

نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.34** Find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.34 using the step-by-step method. **PSV**

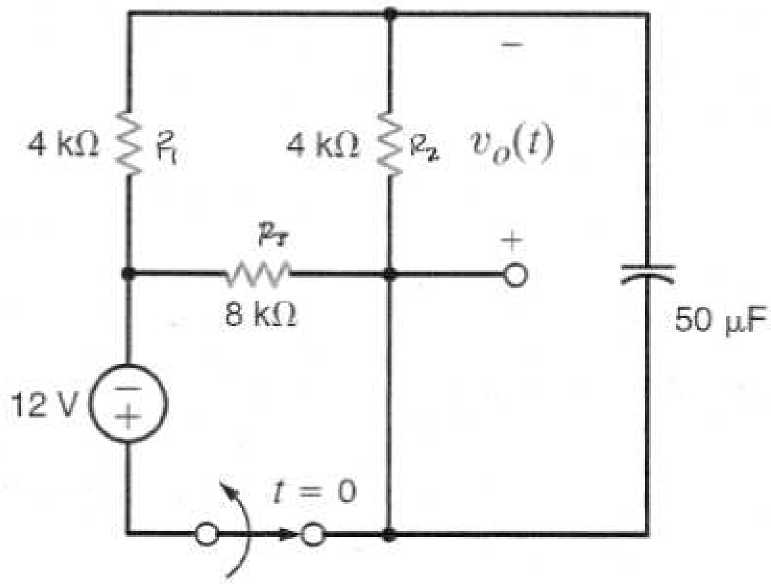
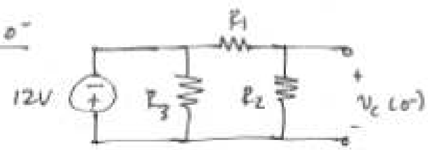



*Figure P7.34*

---

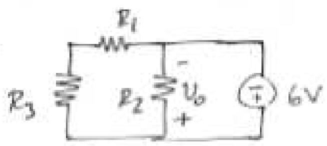
SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$



$v_c(0^-) = -12\left(\frac{R_2}{R_1+R_2}\right) = -6V$

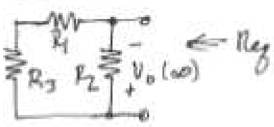
$t = 0^+$



$v_o(0+) = 6V = K_1 + K_2$

$t = \infty$

$v_o(\infty) = 0 = K_1$ $\quad R_{eq} = R_2\ (R_1+R_3)/(R_1+R_2+R_3) = 3\,k\Omega$



$\tau = R_{eq} C = 150 ms$

$\boxed{v_o(t) = 6e^{-6.67t}\ V}$

نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.37** Find $i_o(t)$ for $t > 0$ in the network in Fig. P7.37 using the step-by-step method. CS

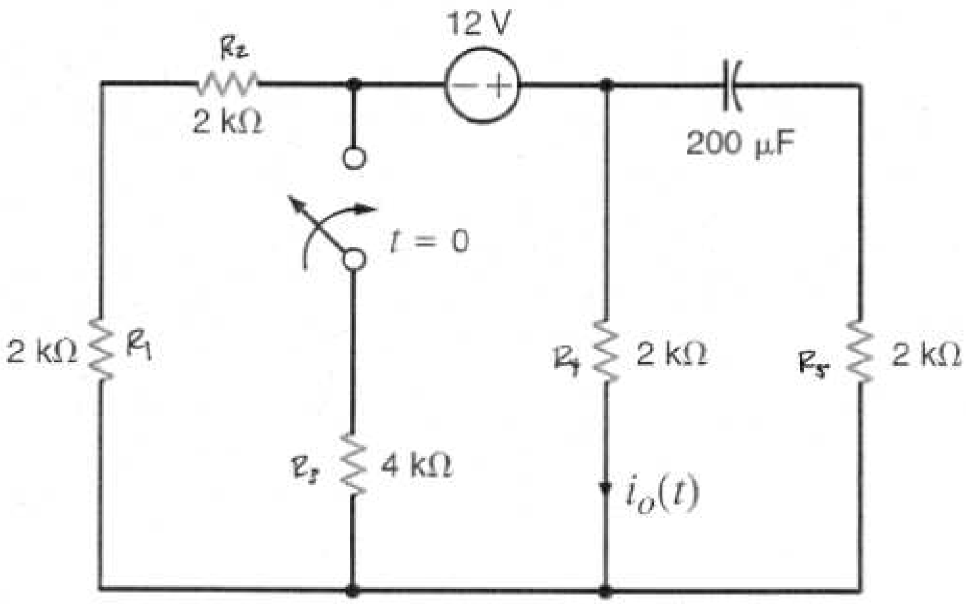
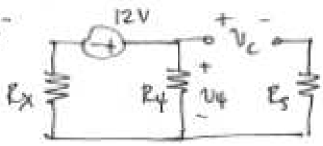



*Figure P7.37*

SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

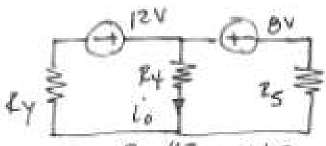
$t = 0^-$




$v_C(0^-) = V_4 = \frac{12 R_4}{R_4 + R_X}$ $\quad R_X = R_1 + R_2 = 4k\Omega$

$v_C(0^-) = 8V$

$t = 0^+$




$\rightarrow$

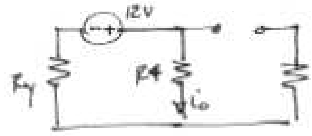

$R_Y = R_X // R_3 = 2k\Omega$


$R_Z = R_Y // R_5 = 1k\Omega$

$i_o(0^+) = \frac{10^{-2} R_Z}{R_Z + R_4} = 3.33\ mA = K_1 + K_2$

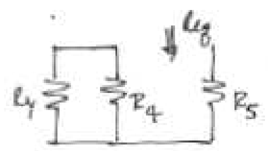
# نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

$\underline{t=\infty}$




$$i_o(\infty) = \frac{+12}{R_\gamma + R_4} = +3mA = K_1$$

$\underline{\tau = ?}$




$$R_{eq} = R_5 + (R_4 /\!/ R_\gamma) = 3k\Omega$$

$$\tau = R_{eq} C = 0.6s$$

$$\boxed{i_o(t) = 3 + 0.33 e^{-1.67t} \quad mA}$$

# نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.38** Use the step-by-step technique to find $i_o(t)$ for $t > 0$ in the network in Fig. P7.38.

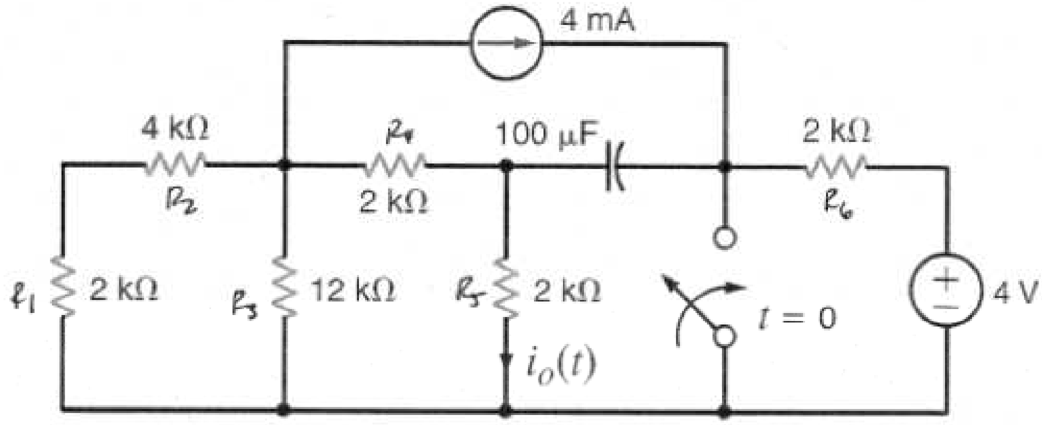



***Figure P7.38***

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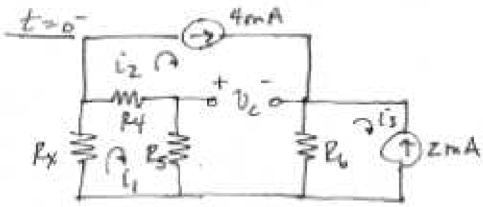
SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$




$R_x = R_3 // (R_1 + R_2) = 4k\Omega$

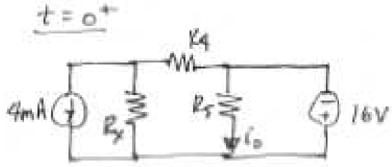
$i_2 = 4mA \qquad i_3 = -2mA$

$i_1(R_x + R_4 + R_5) - i_2(R_4 + R_5) = 0$

$i_1 = 2mA$

$v_C(0^-) = (i_1 - i_2)R_5 + (i_3 - i_2)R_6 = -16\ V$

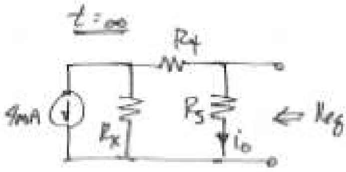
$t = 0^+$




$i_o(0^+) = -\frac{16}{R_5} = -8mA$

$K_1 + K_2 = -8mA$

$t = \infty$




$i_o(\infty) = \frac{-4\times10^{-3} R_x}{R_x + R_4 + R_5}$

$i_o(\infty) = -2mA = K_1$

$\tau = ?$

$R_{eq} = R_5 // (R_4 + R_x)$
$= 1.5k\Omega$

$\tau = R_{eq} C = 0.15 s$

$$\boxed{i_o(t) = -2 - 6e^{-6.67t}\ mA}$$

# نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.19** In the network in Fig. 7.19, find $i_o(t)$ for $t > 0$ using the differential equation approach. CS

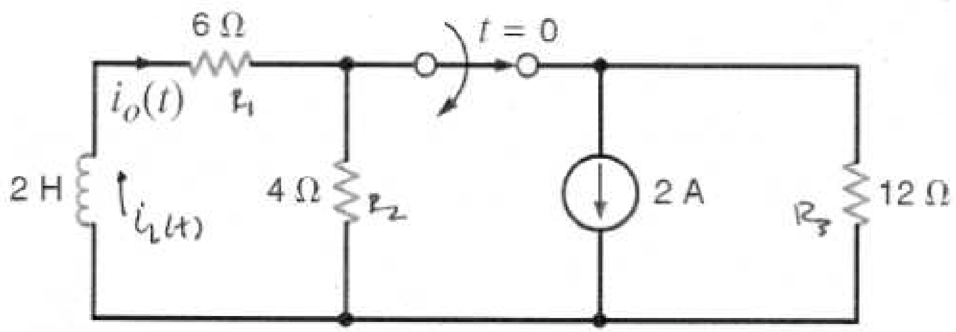



*Figure P7.19*

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SOLUTION: $\underline{t=0^-}$: $\quad i_L(0^-) = \dfrac{2\left(1/R_1\right)}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{2}{3}\text{ A}$

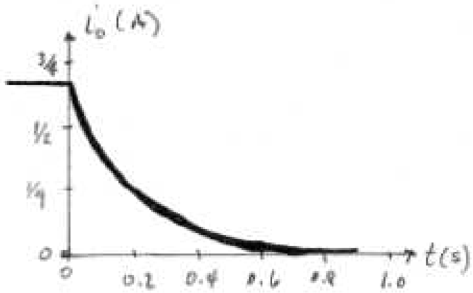
$\underline{t=0^+}$ $\quad i_o = i_L = \frac{2}{3}\text{ A}$

$\underline{t>0}$ $\quad L\dfrac{di_L}{dt} + i_o(R_1+R_2) = 0$ & $i_L = i_o \Rightarrow \dfrac{di_o}{dt} + \dfrac{(R_1+R_2)}{L} i_o = 0$

$i_o = K_1 + K_2 e^{-t/\tau} \Rightarrow \tau = \dfrac{L}{R_1+R_2} = \frac{1}{5}\text{ s} \quad K_1 = 0$

$K_2 = i_o(0^+) - K_1 = \frac{2}{3}\text{ A}$

$$\boxed{i_o(t) = 0.67 e^{-5t}\text{ A}}$$



## نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.20** Use the differential equation approach to find $i(t)$ for $t > 0$ in the circuit in Fig. P7.20 and plot the response including the time interval just prior to switch movement. **PSV**

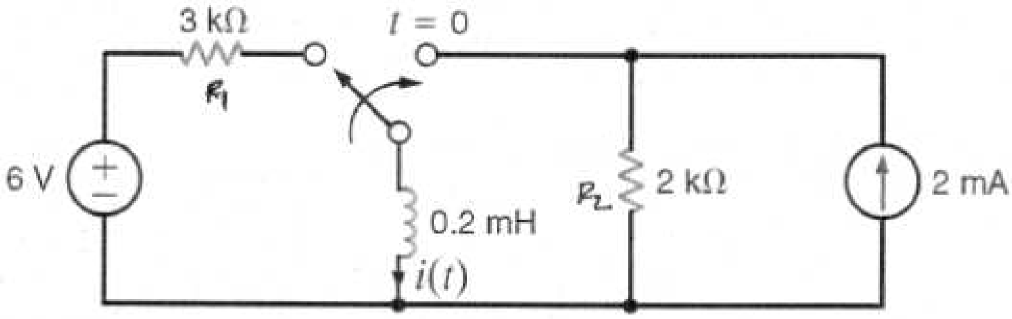



*Figure P7.20*

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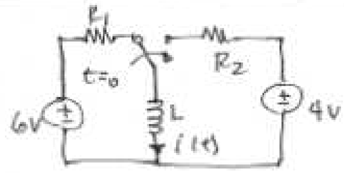
SOLUTION:




$t = 0^-$ : $i(0^-) = \frac{6}{R_1} = 2mA = i(0^+)$

$t = 0^+$ $i(0^+) = 2mA$

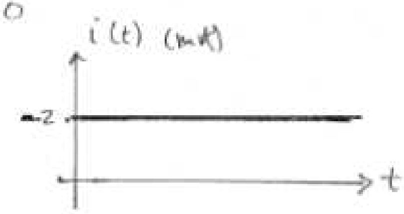
$t > 0$ $4 = R_2 i + L\, di/dt$

$$di/dt + \frac{R_2}{L} i - \frac{4}{L} = 0 \qquad i = K_1 + K_2 e^{-t/\tau}$$

$$\tau = L/R_2 = 0.1\,\mu s \qquad K_1 = 4/R_2 = 2mA$$

$$K_2 = i(0^+) - K_1 = 0$$

$\boxed{i(t) = 2mA}$



# نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.22** Use the differential equation approach to find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.22 and plot the response including the time interval just prior to opening the switch.

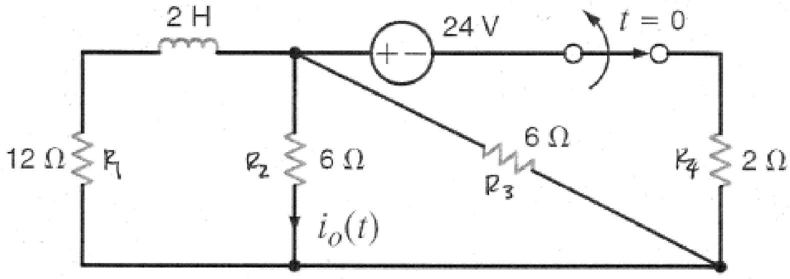



***Figure P7.22***

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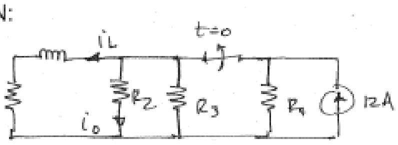
SOLUTION:

$t = 0^-$:



$$i_L(0^-) = \frac{12\,(1/R_1)}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}} = \frac{12}{11}\text{ A}$$

$$i_o(0^-) = \frac{12\text{A}\,(1/R_2)}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}} = \frac{24}{11}\text{ A}$$

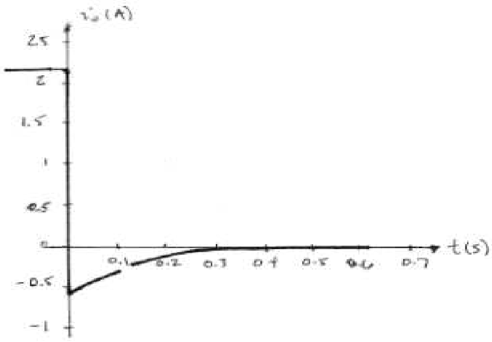
$t = 0^+$ $\quad i_L(0^+) = \frac{12}{11}\text{ A} \qquad i_o(0^+) = -\frac{i_L(0^+)\,R_3}{R_2+R_3} = -\frac{6}{11}\text{ A}$

$t > 0$: $\quad L\frac{di_L}{dt} + i_L\,(R_1+R_B) = 0 \qquad R_B = R_2 // R_3 \qquad i_o = -\frac{i_L R_3}{R_2+R_3}$

$$\frac{di_o}{dt} + \left(\frac{R_1+R_B}{L}\right) i_o = 0 \qquad \& \quad i_o(t) = K_1 + K_2 e^{-t/\tau}$$

$$\tau = \frac{L}{R_1+R_B} = \frac{2}{15}\text{ s} \qquad K_1 = 0 \qquad K_2 = i_o(0^+) - K_1 = -\frac{6}{11}\text{ A}$$

$$\boxed{i_o(t) = -0.545\,e^{-7.5t}\text{ A}}$$



# نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.25** Use the differential equation approach to find $i(t)$ for $t > 0$ in the circuit in Fig. P7.25 and plot the response including the time interval just prior to opening the switch.

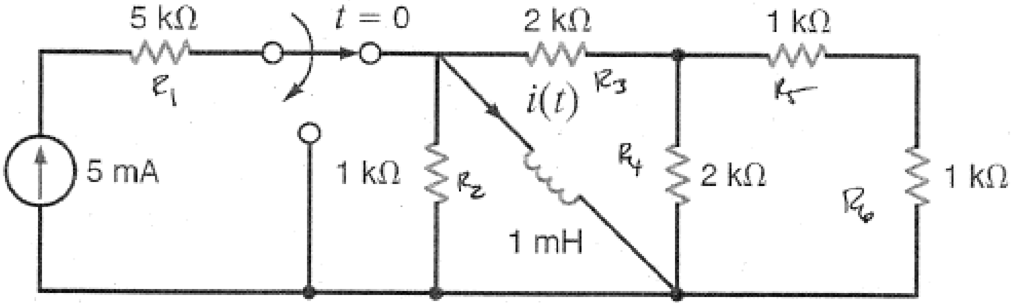



*Figure P7.25*

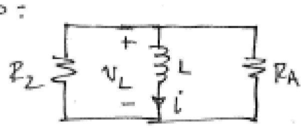
---

SOLUTION:

$t = 0^-$: $i(0^-) = 5\,mA$

$t = 0^+$: $i(0^+) = i(0^-) = 5\,mA$

$t > 0$:




$$i + \frac{v_L}{R_2} + \frac{v_L}{R_A} = 0 \quad \& \quad v_L = L\frac{di}{dt}$$

$$\frac{di}{dt} + \frac{R_A R_2}{(R_A + R_2)L}\, i = 0$$

$$R_A = R_3 + \{R_4 // [R_5 + R_6]\}$$

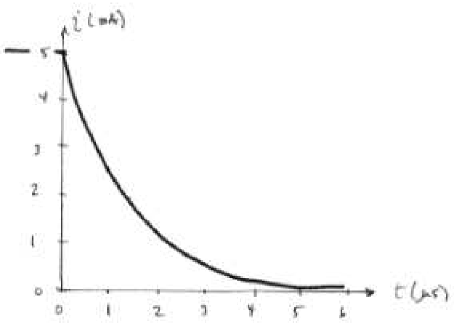
$$R_A = 3\,k\Omega$$

$$i = K_1 + K_2 e^{-t/\tau}$$

$$\tau = \frac{L(R_A + R_2)}{R_A R_2} = \frac{4}{3}\,\mu s$$

$$K_1 = 0 \qquad K_2 = i(0^+) - K_1 = 5\,mA$$

$$\boxed{i(t) = 5e^{-7.5\times10^5 t}\ mA}$$



نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.39** Use the step-by-step method to find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.39.

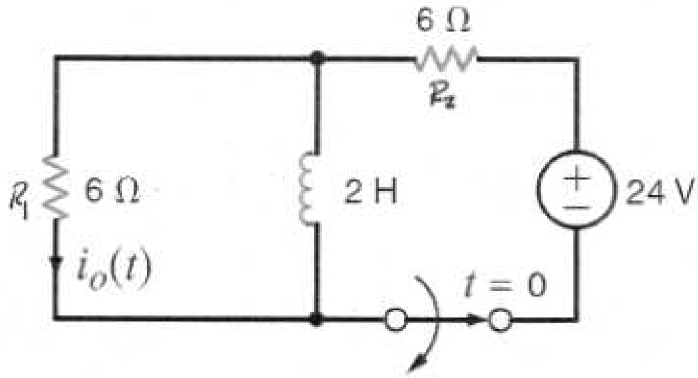



*Figure P7.39*

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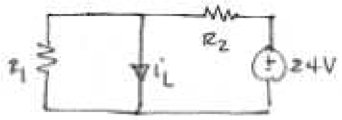
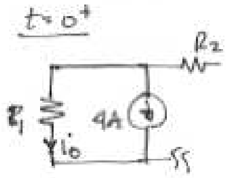
**SOLUTION:** $i_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$




$i_L(0^-) = 24/R_2 = 4A$

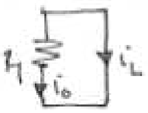
$t = 0^+$




$i_o(0^+) = -4A$

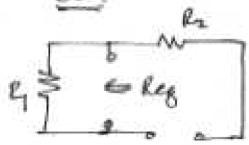
$K_1 + K_2 = -4A$

$t = \infty$




$i_o(\infty) = 0 = K_1$

$\tau = ?$




$R_{eq} = R_1 = 6\Omega$

$\tau = L/R_{eq} = \frac{1}{3}$ s

$\boxed{i_o(t) = -4e^{-3t} \text{ A}}$

## نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.41** Find $i_o(t)$ for $t > 0$ in the network in Fig. P7.41 using the step-by-step method. CS

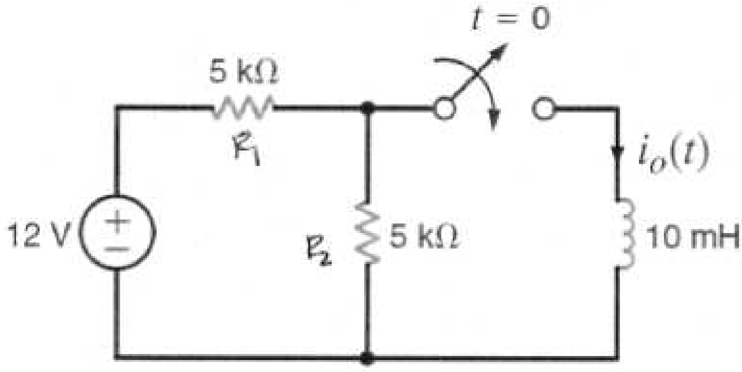



*Figure P7.41*

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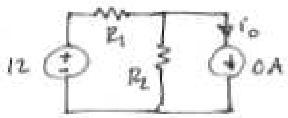
SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

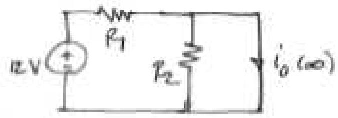
$t = 0^-$


$t = 0^+$



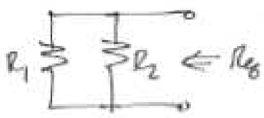

$i_o(0^+) = 0 = K_1 + K_2$

$t = \infty$




$i_o(\infty) = \frac{12}{R_1} = 2.4\,mA = K_1$

$\tau = ?$




$R_{eq} = R_1 // R_2 = 2.5\,k\Omega$

$\tau = \frac{L}{R_{eq}} = 4\,\mu s$

$$\boxed{i_o(t) = 2.4 - 2.4 e^{-2.5\times10^5 t} \; mA}$$

# نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.42** Find $v_o(t)$ for $t > 0$ in the network in Fig. P7.42 using the step-by-step method.

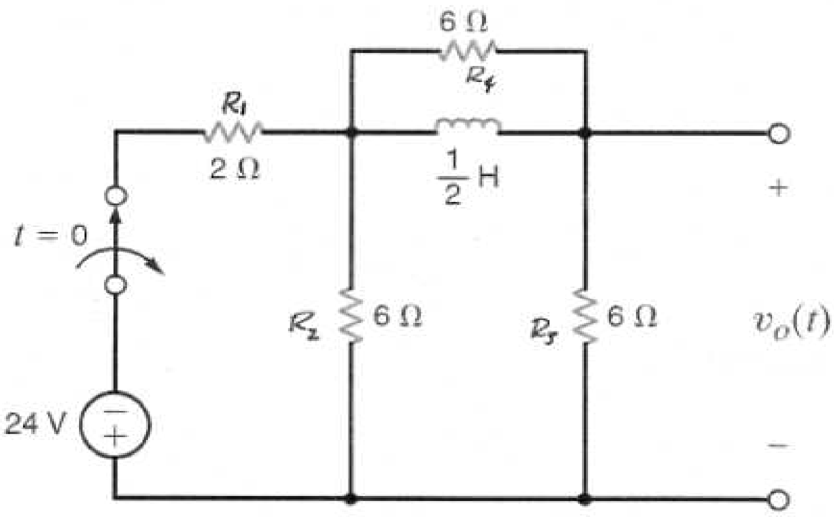



*Figure P7.42*

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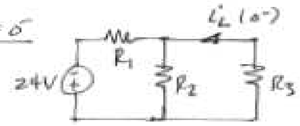
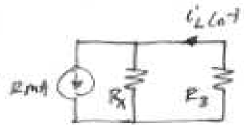
SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$



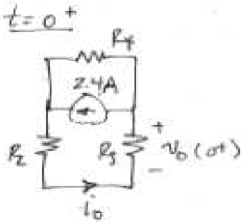

$\Rightarrow$




$R_x = R_1 // R_2 = 1.5k\Omega$

$i_L(0^-) = \frac{12\times10^{-3} R_x}{R_x + R_3} = 2.4A$

$t = 0^+$

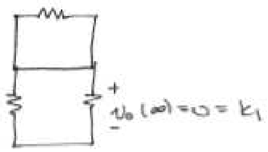



$i_o = \frac{2.4\, R_4}{R_4 + R_3 + R_2} = 0.8\ mA$

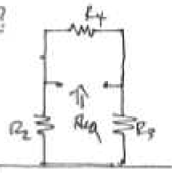
$v_o(0^+) = -R_3\, i_o(0^+) = -4.8V = K_1 + K_2$

# نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

$\underline{t=\infty}$




$\underline{\tau=?}$




$R_{eq} = R_4 \,//\, (R_2+R_3) = 4\,\Omega$

$\tau = L/R_{eq} = \frac{1}{8}\,s$

$$\boxed{v_o(t) = -4.8e^{-8t}\ V}$$

# نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.43** Use the step-by-step method to find $v_o(t)$ for $t > 0$ in the network in Fig. P7.43. **PSV**

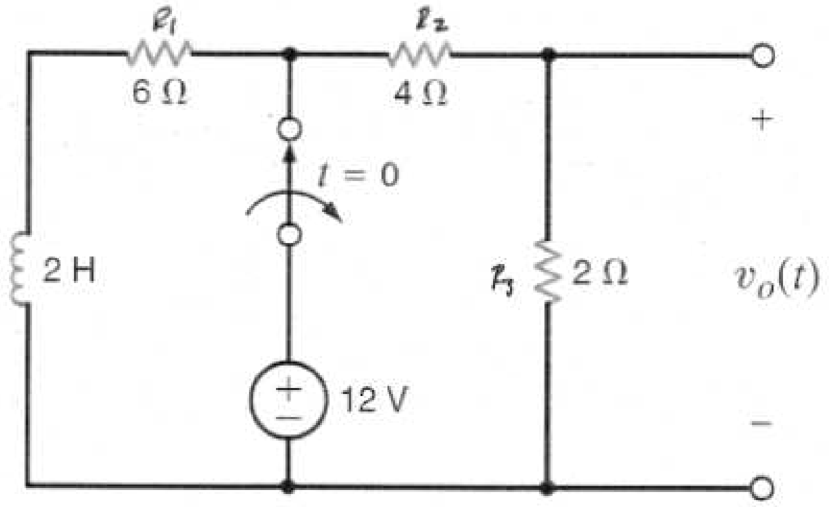
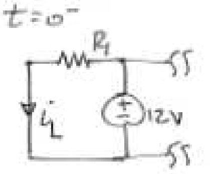



***Figure P7.43***

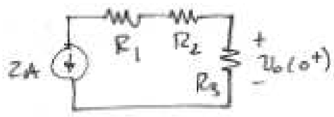
SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$



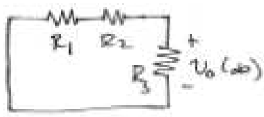

$i_L(0^-) = \frac{12}{R_1} = 2A$

$t = 0^+$



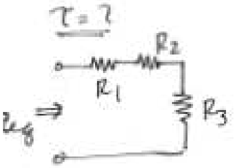

$v_o(0^+) = -2R_3 = -4V$
$= K_1 + K_2$

$t = \infty$




$v_o(\infty) = 0 = K_1$

$\tau = ?$




$R_{eq} = R_1 + R_2 + R_3 = 12\,\Omega$

$\tau = L/R_{eq} = \frac{1}{6}\,s$

$$\boxed{v_o = -4e^{-6t}\ V}$$

نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.48** Find $v_o(t)$ for $t > 0$ in the network in Fig. P7.48 using the step-by-step technique.

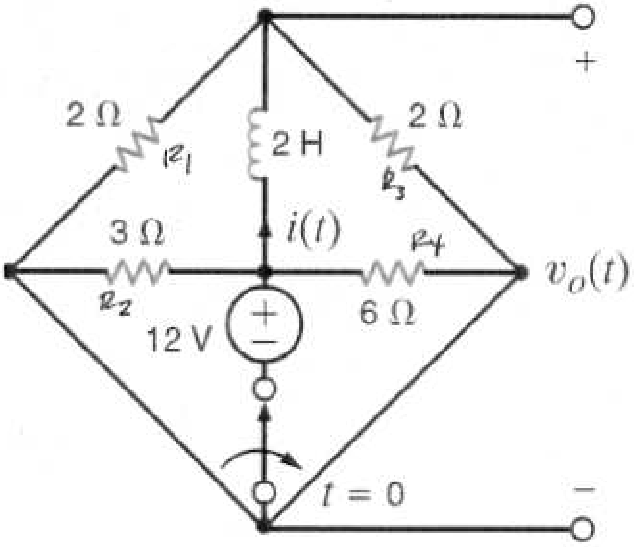



***Figure P7.48***

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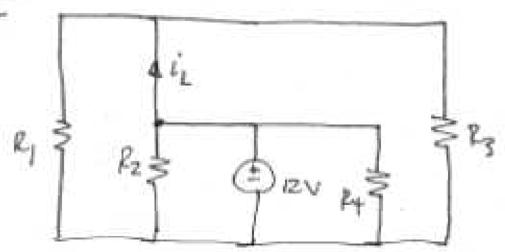
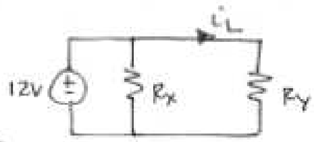
SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$




⇒

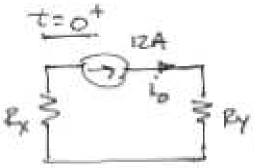


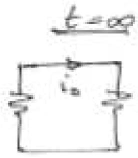

$R_x = R_2 // R_4 = 2\Omega$

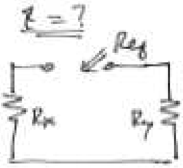
$R_y = R_1 // R_3 = 1\Omega$

$i_L = \frac{12}{R_y} = 12\,A$

## نمونه مسائل حل شده از مبحث مدارهای مرتبه اول




$R_{eq} = R_x + R_y$

$R_{eq} = 3\,\Omega$

$\tau = \frac{L}{R_{eq}} = \frac{2}{3}\ s$

$i_0 = 12A = K_1 + K_2$

$i_0 = 0 = K_1$

$$\boxed{i_0 = 12e^{-1.5t}\ A}$$

# نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.49** Use the step-by-step method to find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.49.

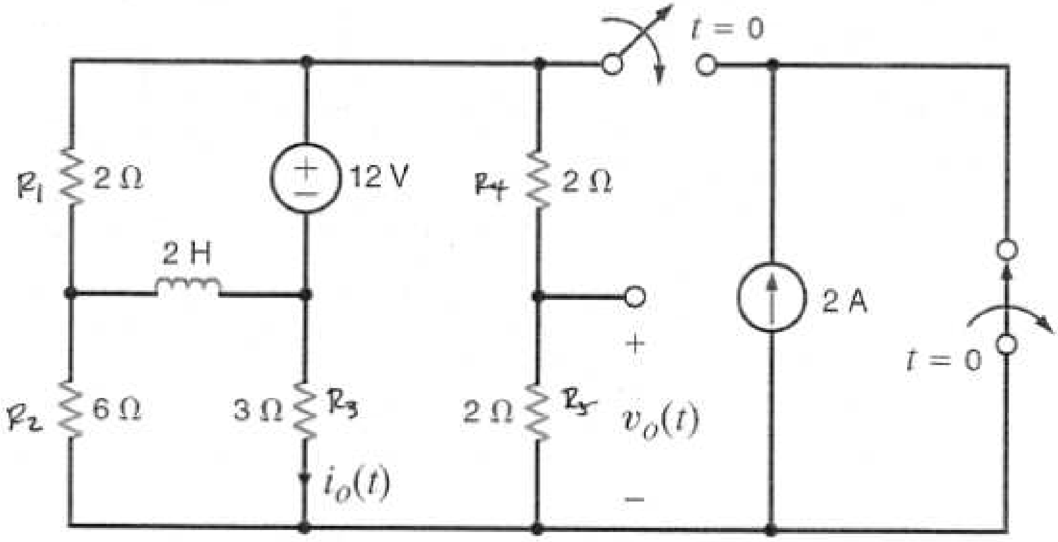



***Figure P7.49***

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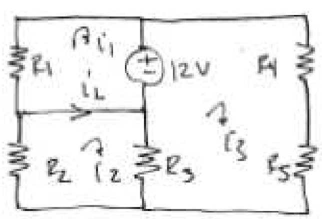
SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$




mesh analysis:

$i_1 R_1 + 12 = 0 \Rightarrow i_1 = -6A$

$i_2(R_2+R_3) - i_3 R_3 = 0 \Rightarrow i_3 = 3 i_2$

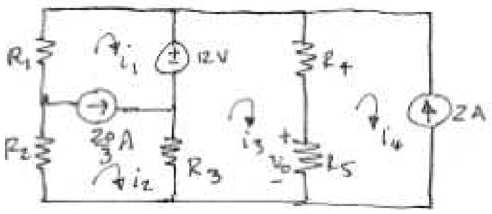
$12 = i_3(R_3+R_4+R_5) - i_2 R_3 \Rightarrow 7 i_3 - 3 i_2 = 12$

yields $i_3 = 2A$ & $i_2 = \frac{2}{3}$ A

$i_L(0^-) = i_2 - i_1 = \frac{20}{3}$ A

# نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

$\underline{t=0^+}$




$$i_2 - i_1 = 20/3$$

$$i_4 = -2$$

$$12 = i_3(R_3+R_4+R_5) - i_2 R_3 - i_4(R_4+R_5)$$

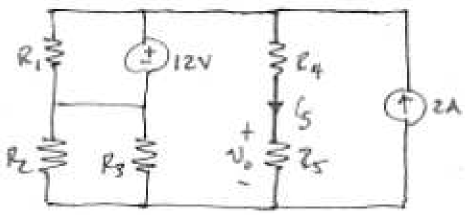
$$0 = i_1 R_1 + i_2 R_2 + i_3(R_4+R_5) - i_4(R_4+R_5)$$

$$i_3 = 0.706\,A$$

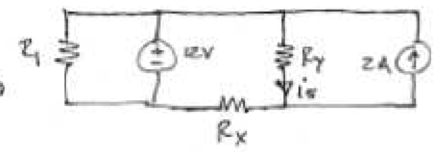
$$v_o = R_5(i_3 - i_4) = 5.41\,V$$

$$5.41 = K_1 + K_2$$

$\underline{t=\infty}$




$\Rightarrow$



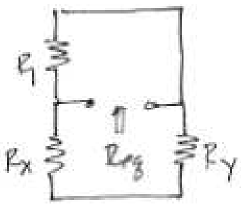

$$R_x = R_2 \parallel R_3 = 2\Omega \qquad R_y = R_4 + R_5 = 4\Omega$$

$$v_0(\infty) = i_5 R_5 = 2 i_5$$

Find $i_5$ by superposition: $i_5 = \dfrac{12}{R_x+R_y} + \dfrac{2R_x}{R_x+R_y} = \dfrac{8}{3}A$

$$v_0 = \frac{16}{3} = 5.33\,V = K_1$$

$\underline{\tau = ?}$




$$R_{eq} = R_1 \parallel (R_x + R_y) = 1.5\Omega$$

$$\tau = L/R_{eq} = \frac{4}{3}\,s$$

$$\boxed{v_0 = 5.33 + 0.08 e^{-0.75t}\ V}$$

# نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.59** Find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.59 using the step-by-step method. CS

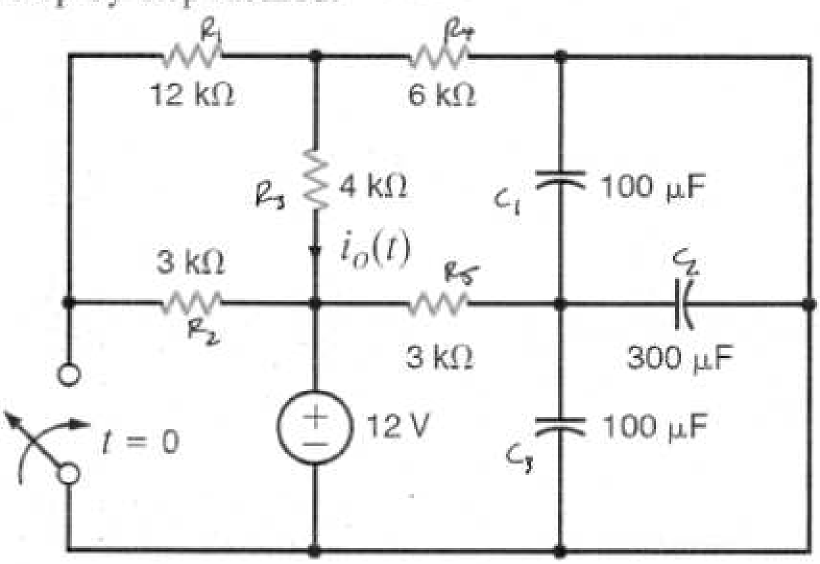



*Figure P7.59*

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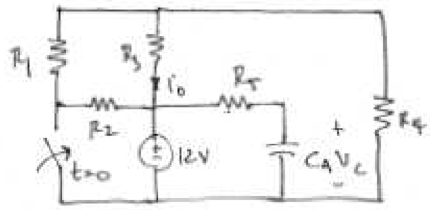
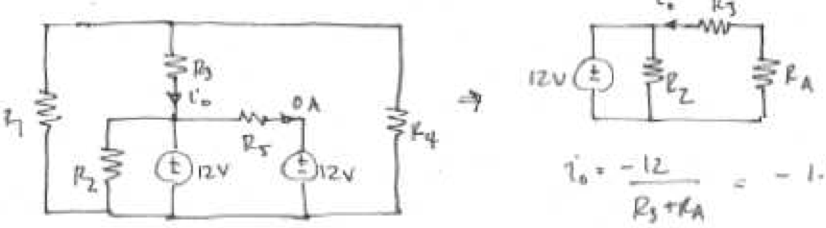
SOLUTION:



$C_A = C_1 + C_2 + C_3 = 500\mu F$

$t = 0^-$: $v_c = 12V$

$t = 0^+$: $v_c = 12V$



$R_A = R_1 // R_4 = 4k\Omega$

$i_o = \frac{-12}{R_3 + R_A} = -1.5mA = K_1 + K_2$

$t = \infty$ Same situation as $t = 0^+$, $i_o = -1.5mA = K_1 \rightarrow K_2 = 0$

$i_o(t) = -1.5mA$

# نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.60** Find $v_o(t)$ for $t > 0$ in the network in Fig. P7.60 using the step-by-step method.

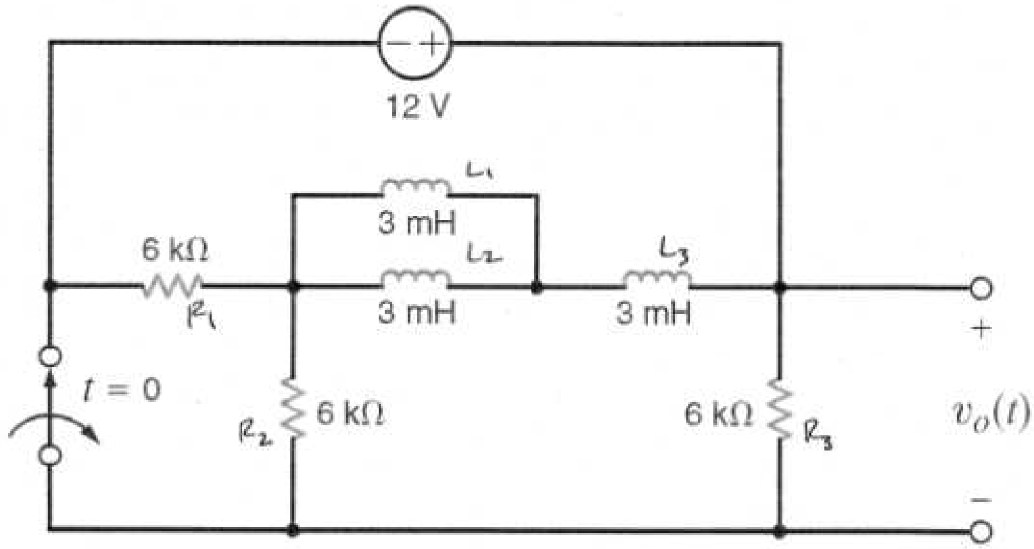



***Figure P7.60***

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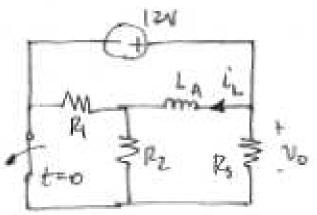
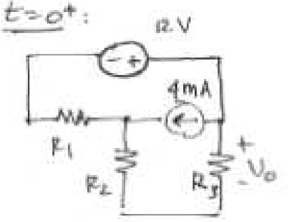
SOLUTION:




$L_A = L_3 + \frac{L_1L_2}{L_1+L_2} = 4.5mH$

$t=0^-$ $\quad i_L = \frac{12}{R_1} + \frac{12}{R_2} = 4mA$

$t=0^+$:

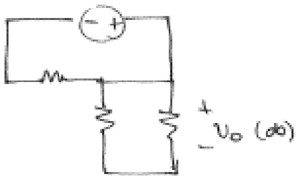



Superposition: $v_o = \frac{12R_3}{R_1+R_2+R_3} - \frac{4\times10^{-3}R_1}{R_1+R_2+R_3}R_3$

$v_o(0^+) - 4V = K_1+K_2$

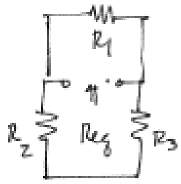
# نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

$t = \infty$ :




$$v_o(\infty) = 0\,V = k_1$$

$\tau = ?$

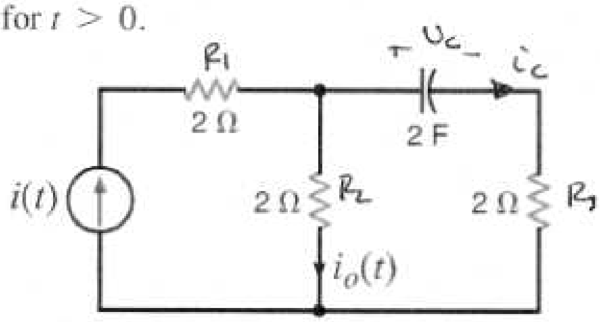



$$R_{eq} = \frac{R_1(R_2+R_3)}{R_1+R_2+R_3} = 4k\Omega \qquad \tau = \frac{L}{R_{eq}} = 1.125\,\mu s$$

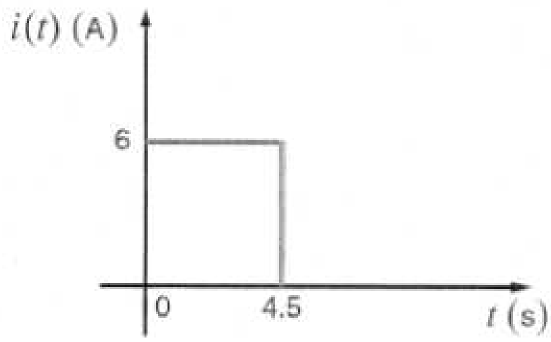
$$\boxed{v_o(t) = -4e^{-8.88\times10^{5}t}\ V}$$

# نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.62** The current source in the network in Fig. P7.62a is defined in Fig. P7.62b. The initial voltage across the capacitor must be zero. (Why?) Determine the current $i_o(t)$ for $t > 0$.




(a)




(b)

***Figure P7.62***

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SOLUTION:

Since $i(t)$ is $\emptyset$ for $t<0$, no charge has accumulated on the capacitor and $v_c$ must be $\emptyset$.

$v_c(t) = K_3 + K_4 e^{-t/\tau}$

$\underline{t=0^-}$ : $v_c = 0$

$i_o(t) = K_1 + K_2 e^{-t/\tau}$

$\underline{t=0^+}$ $\quad v_c = 0 , \quad i_o = \dfrac{i R_3}{R_2 + R_3} = 3A = K_1 + K_2$

# نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

$\underline{t=\infty}$ $\quad i_c = 0A \qquad i_0 = 6 = K_1 \qquad v_c = i_0 R_2 = 12V = K_3$

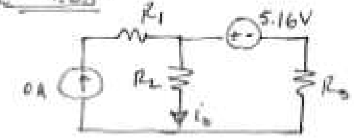
$K_2 = -3A \qquad K_4 = -12V$

$\tau = C R_{eq} = C\,(R_2+R_3) = 8s$

$$\left.\begin{aligned} i_0(t) &= 6 - 3e^{-t/8}\,A \\ v_C(t) &= 12 - 12e^{-t/8}\,V \end{aligned}\right\} \qquad 0 \le t \le 4.5s$$

$\underline{t = 4.5s}$ $\quad v_c(4.5) = 5.16\,V \qquad i_0 = K_5 + K_6 e^{-t'/\tau_2} \qquad t > 4.5s$, $t' = t - 4.5$

$\underline{t = 4.5s^+}$




$i_0 = \dfrac{5.16}{R_2+R_3} = 1.29 = K_5 + K_6$

$\underline{t\to\infty}$ $\quad i_0 = 0 = K_5 \;\Rightarrow\; K_6 = 1.29A$

$\tau_2 = C R_{eq_2} = C\,[R_2+R_3] = 8s$

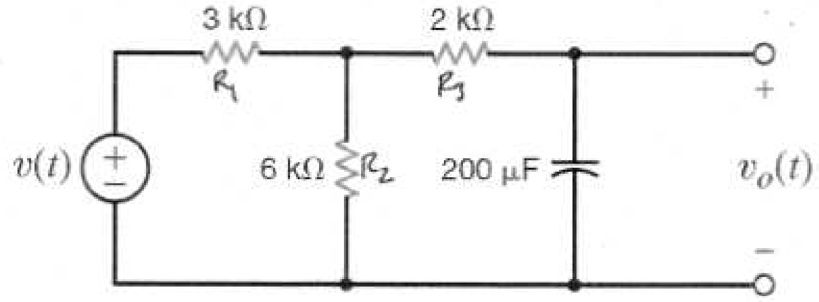
$i_0(t) = 1.29e^{-t'/8}\,A \qquad t > 4.5s$

final answer

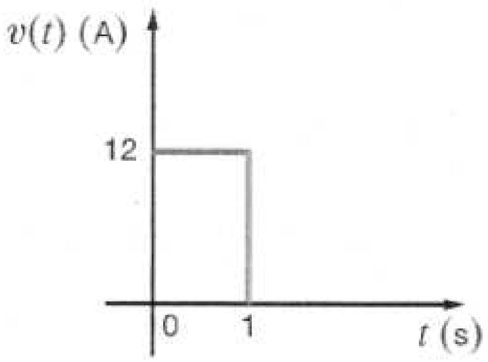
$$\boxed{i_0(t) = \begin{cases} 6 - 3e^{-t/8}\,A & 0 \le t \le 4.5s \\ 1.29e^{-(t-4.5)/8}\,A & t > 4.5s \end{cases}}$$

نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.63** Determine the equation for the voltage $v_o(t)$ for $t > 0$, in Fig. P7.63a when subjected to the input pulse shown in Fig. P7.63b.




(a)



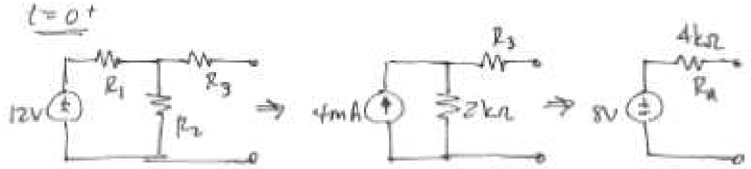

(b)

*Figure P7.63*

SOLUTION: $v_o = K_1 + K_2 e^{-t/\tau}$

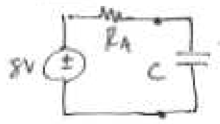
$t=0^-$ $\quad v_o = 0$

$t=0^+$



# نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

$\underline{t=0^+}$




$v_o = v_o(0^-) = 0V = K_1 + K_2$

$\underline{t=\infty}$: $v_o = 8V = K_1$

$\underline{\tau}$ $\quad \tau = CR_A = 0.8s$

$$v_o(t) = 8 - 8e^{-1.25t} \text{ V} \qquad 0 < t \le 1$$

for $t > 1s$, $\quad v_o = K_3 + K_4 e^{-t'/\tau} \qquad t' = t-1$

$\underline{at\ t=1^-}$, $\quad v_o = 5.71\text{ V}$

$\underline{at\ t=1^+}$, $\quad v_o = 5.71\text{V} = K_3 + K_4$

$\underline{at\ t=\infty}$ $\quad v_o = 0 = K_3 \Rightarrow K_4 = 5.71\text{ V}$

$$\boxed{v_o = \begin{cases} 8 - 8e^{-1.25t} \text{ V} & 0 \le t \le 1 \\ 5.71\, e^{-1.25(t-1)} \text{ V} & t > 1 \end{cases}}$$

## نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.64** Find the output voltage $v_o(t)$ in the network in Fig. P7.64 if the input voltage is $v_i(t) = 5\big(u(t) - u(t - 0.05)\big)$ V.

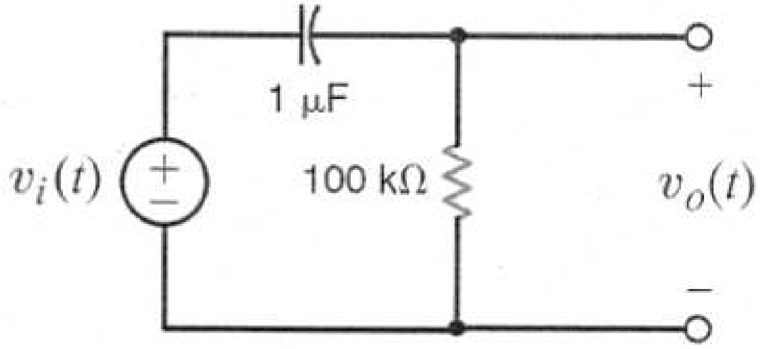



*Figure P7.64*

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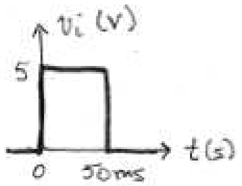
SOLUTION:




For $0 \le t \le 50\text{ms}$ $\quad v_o = k_1 + k_2 e^{-t/\tau}$

For $t > 50\text{ ms}$ $\quad v_o = k_3 + k_4 e^{-t/\tau}$

$t = 0^-$ $\quad v_o = 0$ & $v_c = 0\text{ V}$

$t = 0^+$ $\quad v_c = 0$ & $v_o = v_i = 5 = k_1 + k_2$

$t \to \infty$ $\quad v_o = 0 = k_1 \Rightarrow k_2 = 5\text{ V}$

$\tau = CR = 0.1\text{s} \Rightarrow v_o(t) = 5e^{-10t}$ $\quad 0 \le t \le 50\text{ms}$

at $t = 50\text{ms}^-$ $\quad v_o = 3.03\text{ V}$ & $v_c = 1.97\text{ V}$

$t = 50\text{ms}^+$ $\quad v_c = 1.97\text{ V}$ & $v_o = v_i - v_c = -1.97\text{ V} = k_3 + k_4$

$t \to \infty$ $\quad v_o = 0 = k_4 \Rightarrow v_o(t) = -1.97e^{-10(t-0.05)}\text{ V}$ $\quad t > 50\text{ms}$

$$\boxed{v_o = \begin{cases} 5e^{-10t}\text{ V} & 0 \le t \le 50\text{ms} \\ -1.97e^{-10(t-0.05)}\text{ V} & t > 50\text{ms} \end{cases}}$$

نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.65** The voltage $v(t)$ shown in Fig. P7.65a is given by the graph shown in Fig. P7.65b. If $i_L(0) = 0$, answer the following questions: (a) how much energy is stored in the inductor at $t = 3$ s?, (b) how much power is supplied by the source at $t = 4$ s?, (c) what is $i(t = 6\text{ s})$?, and (d) how much power is absorbed by the inductor at $t = 3$ s?

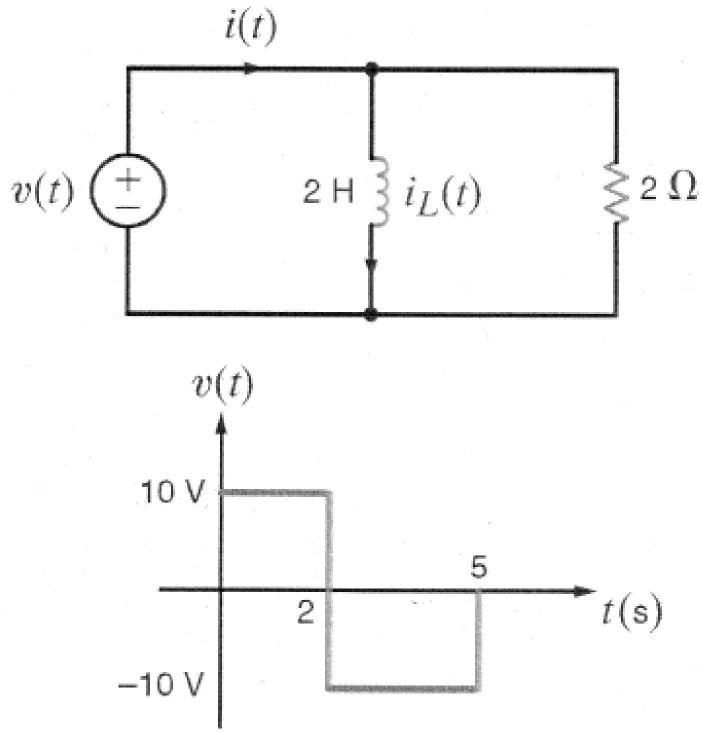



**Figure P7.65**

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SOLUTION:

a) $W_L = \frac{1}{2} L i_L^2$ $\qquad$ $i_L(t) = \frac{1}{L}\int v_L\,dt = \frac{1}{L}\int v\,dt$

$$i_L(3) = \frac{10}{2} t\Big|_0^2 - \frac{10}{2} t\Big|_2^3 = 5\text{A} \qquad \boxed{W_L(3) = 25\text{J}}$$

نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

b) $p_s(t) = v(t)\,i(t) = v(t)\left[i_L(t) + v(t)/R\right]$

$i_L(4) = \frac{1}{L}\int_0^4 v(t)\,dt = 5t\Big|_0^2 - 5t\Big|_2^4 = 0A$

$p_s(4) = v^2(4)/R = 100/2$ $\boxed{p_s(4) = 50W}$

c) $i(6) = i_L(6) + \frac{v(6)}{R}$ $v(6) = 0$

$i_L(6) = \frac{1}{L}\int_0^6 v(t)\,dt = 5t\Big|_0^2 - 5t\Big|_2^5 = -5A$

$\boxed{i(6) = -5A}$

d) $P_L = v(t)\,i_L(t)$ $i_L(3) = 5A$ $v(3) = -10V$

$\boxed{P_L(3) = -50W \text{ absorbed}}$

## نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.66** In the circuit in Fig. P7.66, $v_R(t) = 100e^{-400t}$ V for $t < 0$. Find $v_R(t)$ for $t > 0$.

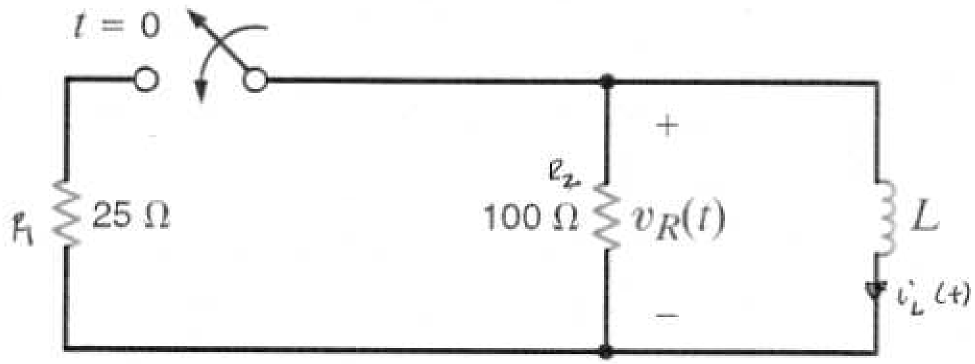



*Figure P7.66*

SOLUTION:

$\underline{t=0^-}$ $\quad v_R(0^-) = 100\,V \qquad i_L(0^-) = -\frac{v_R(0^-)}{R_2} = -1A$

$\tau_1 = \frac{L}{R_2} = \frac{1}{400} \Rightarrow L = \frac{1}{4}\,H$

$\underline{t=0^+}$ $\quad i_L(0^+) = -1A \qquad v_R(0^+) = -\frac{i_L(0^+)\,R_2 R_1}{R_1+R_2} = 20\,V = K_1+K_2$

$\underline{t=\infty}$ $\quad v_R = 0 = K_1 \Rightarrow K_2 = 20\,V$

$\underline{\tau}$ $\quad \tau_2 = \frac{L(R_1+R_2)}{R_1R_2} = \frac{1}{80}\,s$

$$\boxed{v_R(t) = 20e^{-80t}\ V}$$

# نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.67** Given that $v_{C1}(0-) = -10$ V and $v_{C2}(0-) = 20$ V in the circuit in Fig. P7.67, find $i(0+)$.

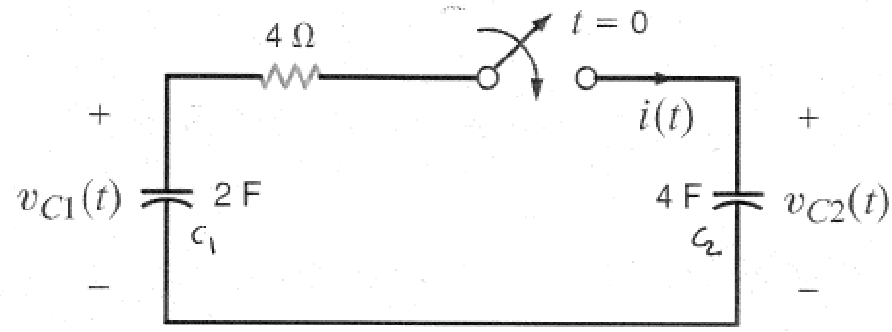
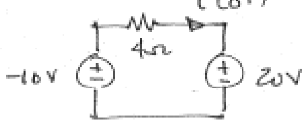



*Figure P7.67*

SOLUTION:

$V_{C1}$ & $V_{C2}$ cannot change instantaneously.

$\underline{t = 0^+}$




$$i(0^+) = \frac{-10 - 20}{4} = -7.5A$$

$$\boxed{i(0^+) = -7.5A}$$

نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.68** The switch in the circuit in Fig. P7.68 is closed at $t = 0$. If $i_1(0-) = 2$ A, determine $i_2(0+)$, $v_R(0+)$, and $i_1(t = \infty)$.

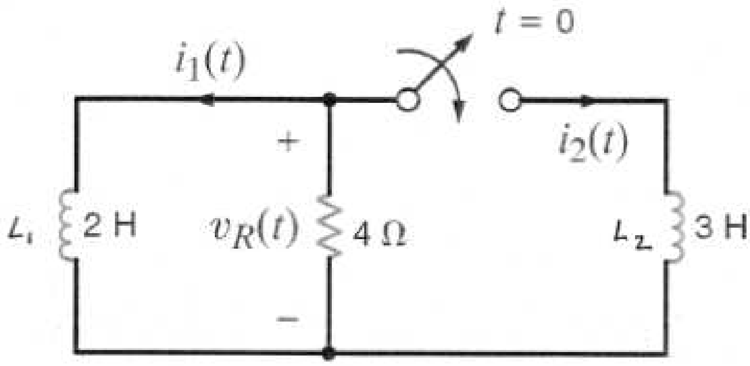



*Figure P7.68*

SOLUTION:

$\underline{t=0^-}$ $\quad i_1(0^-) = 2A \qquad i_2(0^-) = 0A$

$\underline{t=0^+}$ $\quad i_1(0^+) = i_1(0^-) = 2A \qquad i_2(0^+) = i_2(0^-) = 0A$

$v_R(0+) = -i_1(0+)(4) = -8V$

$\boxed{\begin{array}{l} v_R(0+) = -8V \\ i_2(0^+) = 0A \\ i_1(\infty) = 0A \end{array}}$

$\underline{t=\infty}$ all $v(t)$ & $i(t) \to 0$

$i_1(\infty) = 0$

نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.69** In the network in Fig. P7.69 find $i(t)$ for $t > 0$. If $v_{C1}(0-) = -10$ V, calculate $v_{C2}(0-)$.

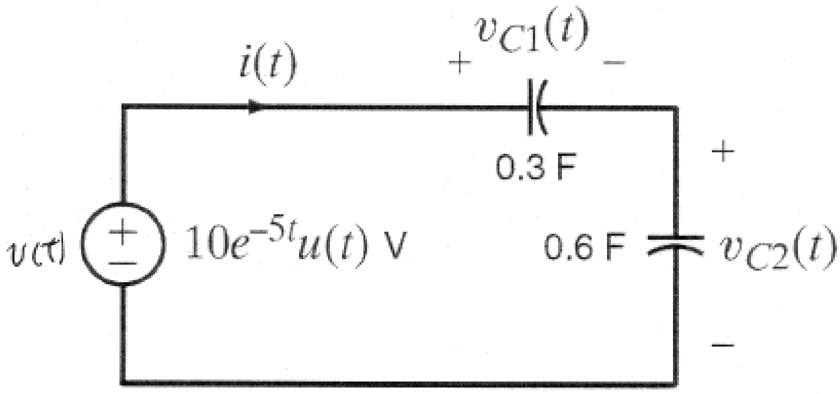
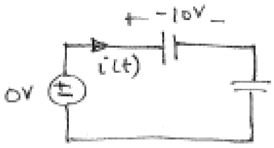



*Figure P7.69*

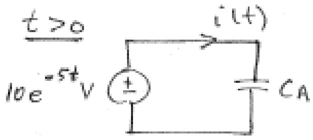
SOLUTION:

at $t = 0^-$



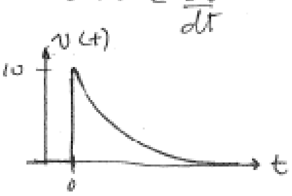

$v_{C2}(0^-) = -v_{C1}(0^-) = 10V$

$t > 0$




$C_A = \frac{C_1 C_2}{C_1 + C_2} = 0.2F$

$i(t) = C\frac{dv}{dt}$




$\frac{dv}{dt} = 10\,\delta(t) - 50e^{-5t} \quad t \geq 0$

$\boxed{\begin{array}{l} i(t) = 2\delta(t) - 10e^{-5t} \text{ A} \quad t \geq 0 \\ v_2(0^-) = 10V \end{array}}$

## نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.71** Given that $i(t) = 13.33e^{-t} - 8.33e^{-0.5t}$ A for $t > 0$ in the network in Fig. P7.71, find the following: (a) $v_C(0)$, (b) $v_C(t = 1\text{ s})$, and (c) the capacitance $C$.

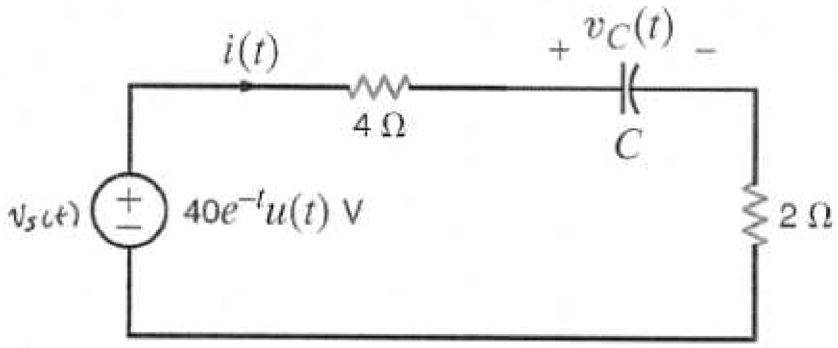



*Figure P7.71*

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**SOLUTION:**

a) $v_c(0^-) = v_s(0^-) = 0\text{V} = v_c(0^+)$ $\quad \boxed{v_c(0) = 0\text{V}}$

b) $v_c(t) = \frac{1}{c}\int i\,dt + K$

$= \frac{1}{c}\left[16.66e^{-t/2} - 13.33e^{-t}\right] + K$

$v_c(0) = 0 = \frac{1}{c}[3.33] + k \Rightarrow k = -3.33/c$

Need C.

c) $\tau = 2 = c[4+2] = 6c \Rightarrow \boxed{c = 1/3\text{ F}}$

Back to b)

$k = -10 \qquad v_c(t) = 50e^{-t/2} - 40e^{-t} - 10$

$\boxed{v_c(1) = 5.61\text{ V}}$

نمونه مسائل حل شده از مبحث مدارهای مرتبه اول

**7.72** Given that $i(t) = 2.5 + 1.5e^{-4t}$ A for $t > 0$ in the circuit in Fig. P7.72, find $R_1$, $R_2$, and $L$.

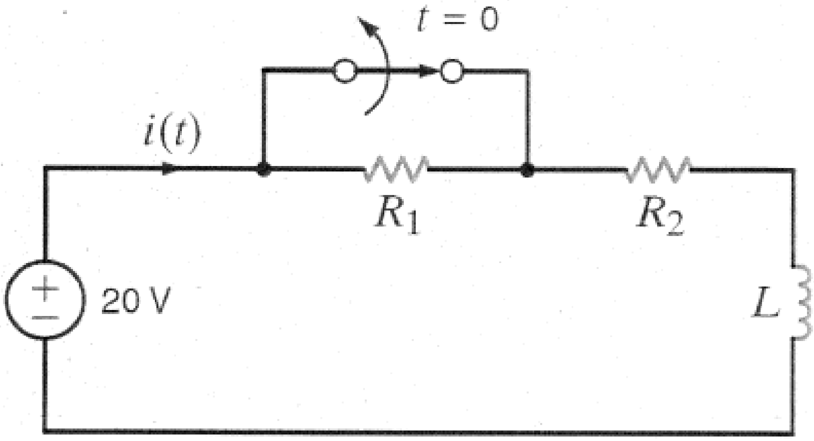


**Figure P7.72**

SOLUTION:

$i(t) = 2.5 + 1.5e^{-4t} = K_1 + K_2 e^{-t/\tau}$

$K_1 = 2.5 = i(\infty) = \frac{20}{R_1+R_2} \Rightarrow R_1 + R_2 = 8\Omega$

$K_1 + K_2 = 4 = i(0^+) = i_L(0^+) = i_L(0^-) = \frac{20}{R_2} \Rightarrow R_2 = 5\Omega$

$R_1 = 3\Omega$

$\tau = \frac{1}{4} = \frac{L}{R_1+R_2}$ $\quad L = 2H$

$$\boxed{\begin{array}{l} L = 2H \\ R_1 = 3\Omega \\ R_2 = 5\Omega \end{array}}$$